Numerical Simulation of Large-Scale Field-Aligned Currents Generation From Finite-Amplitude Magnetosonic Waves

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Two-dimensional numerical simulation of finite-amplitude magnetohydrodynamics (MHD) magnetosonic wave is performed under finite-velocity background convection condition. Isothermal cases are considered for simplicity. External dissipation is introduced by assuming that the field-aligned currents are generated proportional to the accumulated charges. The simulation results are as follows: Paired field-aligned currents are found on the simulated waves. The flow directions of these field-aligned currents depend on the angle between the background convection and the wave normal, and hence two pairs of field-aligned currents are found from a bowed wave as an overall structure. The majority of these field-aligned currents are closed within each pair rather than between two wings. These features are not observed under slow background convection. The result could be applied to cusp current system and the substorm current system.

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1. Introduction

The magnetohydrodynamics (MHD) magnetosonic waves have been barely considered as the source of large-scale field-aligned currents (FACs) because they are independent of the Alfvén mode in a linear homogeneous limit. However, in small scales, twin vortices (i.e., a pair of field-aligned currents) are often observed in the polar region travelling tailward [*Friis-Christensen, et al.*, 1988], indicating that the magnetosonic mode can be coupled with the Alfvén mode [*Southwood*, 1987]. Furthermore, the wave coupling has been intensively investigated in studies of magnetic pulsations [e.g., *Zhu and Kivelson*, 1988, and references therein]. It is known that inhomogenuity such as pressure gradient can cause a coupling between MHD Alfvén waves and magnetosonic waves. However, those results are obtained in a linear harmonic limit. Studies for highly nonlinear applications, e.g., large-scale phenomena remain rather poor.

Recently, *Yamauchi et al.* [1993, hereafter referred to as paper 1] showed analytically that a large-amplitude magnetosonic wave or shock in homogenous plasma can produce paired field-aligned currents when the background convection is sufficiently large or the plasma beta (β) is sufficiently high, i.e., when $\kappa = (1 + 0.5\beta)M > 1$ where M is the Mach number [paper 1]. This is in nonlinear MHD, and is appropriate for large-scale phenomena. The kinetic energy of convection is directly converted to the electrostatic energy that drives the current system, and we call this mechanism "magnetosonic dynamo" hereafter.

The polarity of the paired field-aligned currents depends on the angle between the wave normal and the background convection. If the wave is bowed as shown in Figure 1a, the polarities of the field-aligned currents are different on each wing of the wave. In paper 1, two magnetospheric applications (Figures 1b and 1c) are suggested for such a configuration: 1. the cusp current system [*Iijima and Potemra*, 1976] because the magnetosonic shock is found in the exterior cusp [*Lundin*, 1985]; 2. the substorm current wedge travelling tailward during substorm expansive phase [*Akasofu*, 1964; *McPherron, et al.*, 1973] because paired field-aligned current structures are found inside the poleward-most part of the surge [*Yamauchi et al.*, 1992].

However, one problem is unsolved in paper 1. The current closer direction is unanswered in the bowed configuration in Figure 1. The field-aligned currents can be closed (1) by longitudinal polarisation current (J_P) between the upstream side and downstream side within each wind as well as (2) by the transversal surface current (J_S) between two wings. The latter is the ordinary dynamo circuit whereas the former represents the magnetosonic dynamo. To know the relative importance of these current systems, we need two dimensional (2-D) modelling. This is not achieved by the plane wave approximation in paper 1.

Here we study this by means of 2-D MHD simulation. Since the purpose of this paper is thus limited, we may employ several simplifying assumptions. First we simplify the basic equations to 2-D by neglecting the curvature effect (see paper 1 for more discussion) and the field-aligned flow. Other assumptions are found in paper 1 and in the rest of this paper.

2. Numerical Model

The double field-aligned current solution is analytically derived from the generalized MHD vorticity equation (equation (10) of paper 1), and this can be traced back to a combination of continuity equation and momentum equation, a polytropic relation, frozen-in condition, the Lorentz transform, and a linear current-charge relation (see also *Sato and Iijima* [1979]). Their 2-D forms (under $\mathbf{u}_\parallel = 0$ and $\mathbf{B}\cdot\nabla = 0$) are

$$\frac{\partial}{\partial t}\rho = -\nabla_\perp\cdot(\rho\mathbf{u}) \tag{1a}$$

$$\rho\frac{\partial}{\partial t}\mathbf{u}_\perp = -\rho(\mathbf{u}\cdot\nabla)\mathbf{u}_\perp - \nabla_\perp P + \mathbf{J}_\perp \times \mathbf{B} \tag{1b}$$

$$\frac{\partial}{\partial t}P = -\nabla_{\perp}\cdot(P\mathbf{u}) \tag{1c}$$

$$\frac{\partial}{\partial t}B = -\nabla_{\perp}\cdot(B\mathbf{u}) \tag{1d}$$

$$\mathbf{E}_{\perp} = \mathbf{B}\times\mathbf{u} \tag{1e}$$

$$\nabla_{\perp}\cdot\mathbf{J}_{\perp} = -\sigma\nabla_{\perp}\cdot\mathbf{E}_{\perp} \tag{1f}$$

respectively, where we assumed an isothermal case in (1c) and a uniform conductivity (σ) in (1f), and equation (1f) comes from equation (9) of paper 1.

Equation (1f) is not an explicit form for $\mathbf{J}_{\perp}$, and we have to integrate it for the numerical calculation. Before that, we introduce another assumption: we neglect the magnetic curvature effect and assume $\mathbf{B} = B\hat{z}$ (see Figure 1a for the co-ordinates). This means that the perpendicular components of any vector (e.g., $\mathbf{u}_{\perp}$, $\mathbf{J}_{\perp}$ and $\mathbf{E}_{\perp}$) lie on the (x, y) plane. With this 2-D assumption, the integral form of equation (1f) is written as

$$\begin{aligned}\mathbf{J}_{\perp} - \mathbf{J}_i &= \sigma(\mathbf{u}_{\perp}\times\mathbf{B} - \mathbf{U}_0\times\mathbf{B}_0)\\ &= \sigma(B\mathbf{u}_{\perp}\times\hat{z} - B_0\mathbf{U}_0\times\hat{z})\end{aligned} \tag{2}$$

where $\mathbf{J}_i$ is a divergence-free constant vector, and hence can be expressed in a form of $\mathbf{J}_i = \sigma\nabla_{\perp}\Psi\times\hat{z}$. The reference values U_0 and B_0 (later we also have ρ_0 and P_0) are upstream values of $|u_{\perp}|$, B, ρ, and P without disturbances. Equation (2) is the same as equation (15) of paper 1. The simplest form of the conductivity is a scalar, but in general it is a tensor because of the Hall effect. As long as the non-diagonal component is relatively small, the tensor form does not make the simulation very complicated, and we use a tensor form to have a little more generality. Then, equation (2) becomes

$$\mathbf{J}_{\perp} = (\sigma + \sigma_{\perp}\hat{z}\times)(B\mathbf{u}_{\perp}\times\hat{z} - B_0\mathbf{U}_0\times\hat{z} + \nabla_{\perp}\Psi\times\hat{z}) \tag{3}$$

Now we have to solve Ψ. To do so, we neglect eddy currents which close inside the source region; i.e., we assume that only the potential electric field contributes to $\mathbf{J}_\perp$. That means $\mathbf{J}_i$ gathers all the non-potential part. By taking the total electric field in (3) (i.e., inside the second parenthesis) to be rotation-free, we obtain

$$\nabla_\perp^2 \Psi = -\nabla_\perp \cdot (B\mathbf{u}_\perp) \tag{4}$$

This assumption implicitly removes the magnetic stress force from the momentum equation, and the magnetosonic speed becomes the same as the sound speed in this case. External dissipation is introduced into the system instead of magnetic restore forces. This simplification should not prevent the double field-aligned currents solution from the magnetosonic wave according to the analytical study [paper 1], and is not an inadequate assumption for the present purpose.

For numerical simulation, we need dimensionless forms of equations (1a)-(1d), (3), and (4). Let L_0, a reference value for x and y, be the length of area under consideration. Together with U_0, B_0, ρ_0, and P_0 (already defined above), this consists the basic set of reference values for normalization. For example, the dimensionless time is defined as $t^* = (U_0/L_0)t$. Other reference values are obtained as $\Psi_0 = U_0 B_0$, $J_0 = B_0 \mu^{-1} h^{-1}$, $\sigma_0 = \Sigma_{A0} h^{-1} = h^{-1}\mu^{-1}V_{A0}^{-1}$, where h is the extent of source region along the magnetic field [paper 1]. In the dimensionless forms, two dimensionless parameters, $M_A = U_0 V_{A0}^{-1}$ and $M_S = U_0 C_{S0}^{-1}$, appear in equations (1b) and (3):

$$\frac{\partial}{\partial t^*}(\rho^* u_x^*) + \nabla_\perp^* \cdot (\rho^* u_x^* \mathbf{u}_\perp^*) = -M_S^{-2}\frac{\partial P^*}{\partial x^*} + \frac{L_0}{h} M_A^{-2} J_y^* B^* \tag{5a}$$

$$\frac{\partial}{\partial t^*}(\rho^* u_y^*) + \nabla_\perp^* \cdot (\rho^* u_y^* \mathbf{u}_\perp^*) = -M_S^{-2}\frac{\partial P^*}{\partial y^*} - \frac{L_0}{h} M_A^{-2} J_x^* B^* \tag{5b}$$

$$\mathbf{J}_\perp^* = M_A(\sigma_\perp^* - \sigma^* \hat{z}\times)(B^* \mathbf{u}_\perp^* + \hat{x} + \nabla_\perp^* \Psi^*) \tag{5c}$$

whereas no parameter appears in the dimensionless forms of other equations (1a), (1c), (1d), and (4). Note that the $J_z \mathbf{B}_{x,y}$ term is neglected in equations (5b) and (5b) because we do not include the magnetic curvature effect and because inclusion of these small terms does not alter the solution significantly [*Yamauchi*, 1990]. The field-aligned current $J^*_\parallel = J^*_z(z\!=\!h)$ is then obtained by height-integrating $\nabla_\perp\!\cdot\!\mathbf{J}_\perp$ inside the source region:

$$J^*_\parallel = -\frac{h}{L_0}\nabla^*_\perp\!\cdot\!\mathbf{J}^*_\perp \tag{6}$$

The initial conditions are simple: we consider only a uniform convection without any disturbances corresponding to the upstream of Figure 1a. Therefore, we have $\rho^* = P^* = B^* = 1$, $u^*_x = -1$, and $u^*_y = \Psi^* = J^*_x = J^*_y = 0$. For the boundary conditions, we give a gradual pressure increase on the downstream boundary, the highest pressure enhancement in the centre, and no enhancement in the side edges of the outflow boundary. In this way we may have a bowed compressional wave propagating against the uniform convection. The other three boundaries are free boundaries, i.e., the first spatial derivative is set zero in the normal direction to the boundary surface. The free boundary are inelastic for waves (no reflection at the boundary).

Time evolutionary equations (1a)-(1d) are integrated using the Two Step Lax Wendroff method [*MacCormack*, 1969]. The meshes of the simulation box are 40×40, and the box size is $L_0 \times L_0$. Each time step is determined as $\Delta t^* = 0.25\cdot Min(\Delta x^*, \Delta y^*)\cdot[C^*_S + Max(|V^*|)]^{-1}$ so as to satisfy the Courant's stability condition [*Roache*, 1976]. For the parameter values, we adopt $M_A = 0.45$, $M_S = 0.71$ (i.e., $\beta = 0.8$), $\sigma^* = 0.45$, $\sigma^*_\perp = 0.22$, and $L_0 h^{-1} = 5$. Such values of M_A and M_S are not unrealistic in the central plasma sheet if the magnetospheric convection is as fast as 30–100 $\mathrm{km\,s^{-1}}$ [*Moore et al.*, 1987]. With these values, we have $\kappa = 0.99$ (see introduction for κ).

3. Simulation Results

Figures 2*a* and 2*b* show the numerical results. It is clear from the figures that a finite-amplitude compressional wave is propagating against the convection as is expected from the initial and boundary conditions (subsonic convection plus pressure increase at the downstream boundary). The wave front is characterized by a sudden increase of both the pressure and the magnetic field, indicating that this belongs to the MHD fast mode. The propagation velocity is $0.4U_0$ against the convection, i.e., $1.4U_0$ relative propagation velocity. This value is the same as the sound speed ($C_S = M_S^{-1}U_0 = 1.4U_0$). Since the magnetosonic speed is the same as the sound speed in the present formulation (see discussion at equation (4)), this result supports that the simulated wave is the MHD fast wave. The wave is steepened to form a shock in Figure 2*b*. Since we used the conservation forms in the basic equations, the conservation relations are satisfied across the shock.

Intense field-aligned currents are recognized on the wave front. At first (Figure 2*a*) the field-aligned currents are not in pairs: only one sheet of field-aligned current appears on each wing (top and bottom) of the bowed wave. Later (Figure 2*b*), however, oppositely-directed field-aligned currents appear downstream side of the first ones on both sides, respectively. The current distribution agrees with that of Figure 1a. One important finding is that the current system is mostly closed between the upstream-side and downstream-side rather than between both wings of the bowed wave. Hence, we have intense enough current to explain the large-scale field-aligned currents in the ionosphere. The intensity of those field-aligned currents is more than $0.2B_0\mu^{-1}h^{-1}$, i.e., more than $B_0\mu^{-1}L_0^{-1}$.

The downstream-side field-aligned current is always less intense than the upstream-side one. The asymmetry comes from the Hall term ($\sigma_\perp$). The result becomes symmetric if we set $\sigma_\perp = 0$. In other words, the expected four-current structure is modified but not totally lost by including the Hall effect [*Yamauchi*, 1990].

The simulation is done with different sets of parameter values [*Yamauchi*, 1990], but the main results (wave steepening and formation of paired field-aligned current system) are the same unless $\kappa < 0.8$, and hence we present only this case here. For small κ, i.e., for slow background convection, we do not observe steepened waves nor the paired current system. Finite background convection (finite inflow kinetic energy) is thus crucial for the magnetosonic dynamo to work. The κ dependence also indicates that the wave steepening is not the artifact of dispersion which is inherent to two-step Lax-Wendroff method.

4. Conclusion

It is numerically shown that a bowed large-amplitude magnetosonic wave propagating against the background convection can generate two pairs of field-aligned currents as shown in Figure 1 as is predicted in paper 1. This solution, however, is not obtained for slow background convection. The majority of these field-aligned currents are found to be closed by longitudinal polarization currents within each wing of the bowed wave rather than by the transversal surface current of the wave. Thus, the field-aligned current due to magnetosonic dynamo can be stronger than that by the traditional dynamo, where the field-aligned currents closed by the transversal current. We also confirmed that the solution is valid with finite Hall effect.

In the introduction, we proposed two magnetospheric phenomena for possible applications. However, before we could apply the above result to the substorm current wedge, we have to consider, e.g., the effect of the magnetic curvature and the mechanism to enhance the near-Earth pressure (this generates a magnetosonic wave). Similarly, we have to obtain a standing wave solution before we could apply to the cusp current system. Yet, the magnetosonic mechanism in bowed magnetosonic waves can be still useful in understanding these magnetospheric phenomena.

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Fig. 1. A predicted distribution of the field-aligned currents on a bowed compressional wave propagating against the finite-velocity background convection. (a) General configuration. The transversal surface current (J_S) generally exists for ordinary MHD fast waves but does not diverge in 1-D configuration. The longitudinal current (J_P) is responsible for the paired structure. (b) and (c) Possible magnetospheric regions (the exterior cusp and the substorm current wedge expanding against convection) where the magnetospheric dynamo can be applied (after paper 1).

Fig. 2. Numerical results at (a) 250 time steps ($t^* = 1.03$) and (b) 500 time steps ($t^* = 2.02$). The top left panel shows the pressure P^*, the top right panel shows the velocity $\mathbf{u}_\perp^*$, the bottom left panel shows the field-aligned current J_z^*, and the bottom right panel shows the perpendicular current $\mathbf{J}_\perp^*$. The distributions of $B_z > 0$ and ρ (not shown here) are similar to the pressure distribution. The excited large-scale MHD fast mode is accompanied by the field-aligned currents as is expected.