Vorticity Equation for MHD Fast Waves in Geospace Environment

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The magnetohydrodynamic (MHD) vorticity equation is modified in order to apply it to nonlinear MHD fast waves or shocks when their extent along the magnetic field is limited. Field-aligned current (FAC) generation is also discussed on the basis of this modified vorticity equation. When the wave normal is not aligned to the finite velocity convection and the source region is spatially limited, a longitudinal polarization ($\mathbf{u}_\perp \cdot \mathbf{J}_\perp$) causes a pair of plus and minus charges inside the compressional plane waves or shocks, generating a pair of FACs. This polarization is not related to the separation between the electrons and ions caused by their difference in mass (i.e., Langmuir mode), a separation which is inherent to compressional waves. The resultant double field-aligned current structure exists both with and without the contributions from curvature drift, which is questionable in terms of its contribution to vorticity change from the viewpoint of single-particle motion.

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1. Introduction

The importance of the magnetohydrodynamic (MHD) vorticity equation in studying generation of the large-scale field-aligned current (FAC) has been suggested by many authors [e.g., *Hasegawa and Sato*, 1980; *Vasyliunas*, 1984]. However, the most frequently used form, for example, the one by *Sato and Iijima* [1979], is rather general and can be simplified further if we restrict our discussion to the MHD fast waves or shocks as shown in Figure 1 because special relations exist in the MHD fast mode. Further, Sato and Iijima's vorticity equation is based on more or less stationary plasma, and may not be applicable to a standing structure (an MHD fast shock) in a fast flow or a rapidly moving structure (an MHD fast wave) in Figure 1. The source region of vorticity moves far upstream before the bouncing drift particle (which carries the information of "drift"caused by curvature) can be reflected back from, for example, a magnetic mirror. Hence the curvature drift does not always have to be included. The purpose of this paper is to derive a modified vorticity equation applicable to the above situation.

We also discuss the resulting generation of FACs. Since the vorticity equation can be obtained by the particle drift theory as well as by MHD theory, it is often expressed with the current divergence term on its left-hand side when FAC generation is discussed. This procedure is an appropriate one when the time derivative of the vorticity is not essential, i.e., for rather slow convection or steady state convection [*Wolf and Spiro*, 1985, and references therein]. However, we take a different approach because the present situation is more dynamic. The MHD vorticity equation has its root on Newton's second law as it is equivalent to the momentum equation, and hence it can be interpreted as a cause-effect relation of how the vorticity is generated or deformed due to the other physical quantities. Having the current divergence term on the left hand side possibly makes it difficult to see this cause-effect relation. Therefore we attempt to keep the vorticity term on the left-hand side of the equation. This procedure does not necessarily prohibit the use of the vorticity equation to calculate the divergence of the perpendicular current: one may still discuss the FAC generation using the vorticity equation after the whole set of MHD equations is solved simultaneously, for example, by numerical simulations, which is left for future studies.

2. Modified Vorticity Equation

The fully nonlinear form of the MHD vorticity equation in the magnetosphere can be found in equation (A6) of *Hasegawa and Sato* [1980]:

$$\rho \frac{d}{dt} \frac{(\nabla \times \mathbf{u})_{\parallel}}{B} = -\nabla_{\perp} \cdot \mathbf{J}_{\perp} + \mathbf{J}_{iner} \cdot \frac{\nabla \rho}{\rho} + (\nabla P + \rho \frac{d\mathbf{u}}{dt}) \cdot (\frac{\mathbf{J}_{curv} + \mathbf{J}_{\nabla B}}{P}) \tag{1}$$

where the subscript $\parallel$ represents parallel component to the magnetic field, and the subscripts curv, ∇B, and iner represent the contributions from the curvature drift, the gradient B drift, and the inertia drift, respectively. Equation (1) is the original form of what *Sato and Iijima* [1979] used. Note that the sign for the $\nabla \rho$ term should be plus instead of minus as appeared in Hasegawa and Sato's equation (A6). Note also that there is another misprint in *Sato and Iijima*'s [1979] equation (4) (∇B should be ∇B^2).

Let us consider each drift's contribution inside an MHD fast wave propagating across the background magnetic field. We assume that the extent of this wave along the magnetic field is limited as shown in Figure 1. In this case, any particle drift directly related to the gyromotion must be still taken into account, while a particle drift associated with longitudinal motion (like mirror bouncing) may not always contribute to the vorticity equation because the wave is expected to move further upstream before the mirror-bounced particles (which eventually carry the information of the "drift") come back to the wave. Among the three types of drifts in equation (1), the curvature drift involves the longitudinal motion. Thus we may not expect that the curvature drift always contributes to the vorticity equation (1) when a height-limited wave is moving across the magnetic field as shown in the Figure 1.

On the basis of the above consideration, we propose to use the following equation instead of (1):

$$\rho \frac{d}{dt} \frac{(\nabla \times \mathbf{u})_{\|}}{B} = - \nabla_{\perp} \cdot \mathbf{J}_{\perp} + \mathbf{J}_{iner} \cdot \frac{\nabla \rho}{\rho} + (\mathbf{J}_{\perp} \times \mathbf{B}) \cdot \frac{(\alpha \mathbf{J}_{curv} + \mathbf{J}_{\nabla B})}{P} \tag{2}$$

where we used the momentum equation to rewrite $\nabla P + \rho d\mathbf{u}/dt$. The only difference between (1) and (2) is the additional parameter α ($0 \leq \alpha \leq 1$) on the $\mathbf{J}_{curv}$ term. We have $\alpha = 1$ if the curvature drift is fully included and $\alpha = 0$ if it is totally neglected. Note that the curvature drift term is derived without taking into account the mirror bouncing; yet physical contributors of this drift term are bounced particles. Therefore we now leave both possibilities of having $\alpha = 1$ and $\alpha < 1$.

By inserting the expression for $\mathbf{J}_{curv}$, $\mathbf{J}_{\nabla B}$, and $\mathbf{J}_{iner}$, i.e.,

$$\frac{\mathbf{J}_{curv}}{P} = \frac{\nabla \times \hat{\mathbf{b}}}{B} \tag{3a}$$

$$\frac{\mathbf{J}_{\nabla B}}{P} = \nabla(\frac{1}{B}) \times \hat{\mathbf{b}} \tag{3b}$$

$$\mathbf{J}_{iner} = \frac{\hat{\mathbf{b}}}{B} \times (\rho \frac{d\mathbf{u}}{dt}) = \mathbf{J}_{\perp} - \frac{\hat{\mathbf{b}} \times \nabla P}{B} \tag{3c}$$

equation (2) can be rewritten as

$$\frac{d}{dt} \frac{(\nabla \times \mathbf{u})_{\|}}{B} = - \frac{\nabla_{\perp} \cdot \mathbf{J}_{\perp}}{\rho} - (1 + \alpha) \frac{\mathbf{J}_{\perp}}{\rho B} \cdot \nabla B + \mathbf{J}_{\perp} \cdot \frac{\nabla \rho}{\rho^2} - \frac{(\nabla P \times \nabla \rho)_{\|}}{\rho^2 B} \tag{4}$$

where $\hat{\mathbf{b}} = \mathbf{B}/B$. It is now obvious from (3) that neglecting the curvature drift is equivalent to taking a two-dimensional approximation (all quantities are uniform along $\mathbf{B}$), which is appropriate for plane MHD fast wave or shock propagating across the geomagnetic field.

Let us consider possible additional relations that the MHD fast mode may obey in the configuration of Figure 1. The following two assumptions may be appropriate for this situation:

$$\nabla P \parallel \nabla \rho \tag{5a}$$

$$B \propto \rho \tag{5b}$$

Assumption (5a) is an extension of the polytropic relation $d(P\rho^{-\gamma})/dt = 0$ and assumption (5b) in an extension of the frozen-in condition $d(B\rho^{-1})/dt = 0$. Strictly speaking, these conditions are valid only within the same fluid element, but we extend them to the entire two-dimensional plane perpendicular to the magnetic field. This extension is not a bad assumption as long as we consider a plane MHD fast wave propagating across the magnetic field. Although this assumption is not always valid (e.g., assumption (5b) is violated whenever the diamagnetic effect is essential like MHD slow waves), there are situations where conditions (5a) and (5b) are satisfied.

Under these assumptions, equation (4) becomes much simpler:

$$\frac{d}{dt}\,\frac{(\nabla\times\mathbf{u})_{\parallel}}{B} = -\frac{\nabla_{\perp}\cdot\mathbf{J}_{\perp}}{\rho} - \alpha\frac{\mathbf{J}_{\perp}\cdot\nabla B}{\rho B} \tag{6}$$

This is the vorticity equation for the MHD fast waves or shocks. The value of α depends on the configuration along the magnetic field; i.e., it is unity if the extent of the wave is infinite along the magnetic field whereas it is less than unity if the wave is "height-limited" as shown in Figure 1.

Equation (6) tells us how the vorticity is generated from given self-consistent distributions of B, ρ, $\mathbf{J}_{\perp}$, etc.; however, this equation alone is incomplete for determining the generation of FACs. This can be seen by expressing the vorticity equation in terms of the convection electric field.

$$\nabla_{\perp}\cdot\mathbf{E}_{\perp} = -B(\nabla\times\mathbf{u})_{\parallel} + \mathbf{u}_{\perp}\cdot\nabla\times\mathbf{B} \tag{7}$$

Note that the parallel electric field is zero inside the source region because of the frozen-in condition (5b). With the help of (7), equation (6) becomes

$$\frac{d}{dt}\frac{\nabla_\perp\cdot\mathbf{E}_\perp}{B^2}-\frac{B^2}{\rho}\frac{\nabla_\perp\cdot\mathbf{J}_\perp}{B^2}=\frac{d}{dt}\frac{\mathbf{u}_\perp\cdot\nabla\times\mathbf{B}}{B^2}+\alpha\frac{\mathbf{J}_\perp\cdot\nabla B}{\rho B} \tag{8}$$

It is now clear from equation (8) that the determination of the field-aligned current generation requires the $\mathbf{J}_\perp$-$\mathbf{E}_\perp$ relation to be specified. In other words, we need to specify how the actual FACs are generated from given space charges.

3. Possible J-E Relations

The most probable relation between the space charges $Q_C = h\epsilon_0\nabla_\perp\cdot\mathbf{E}_\perp$ and the field-aligned current $J_\| = -h\nabla_\perp\cdot\mathbf{J}_\perp$ is a linear relation; i.e., $J_\| \propto Q_C$ or

$$\nabla_\perp\cdot\mathbf{J}_\perp = -\sigma\nabla_\perp\cdot\mathbf{E}_\perp \tag{9a}$$

where h is the height of the source region along the magnetic field and σ is a proportional constant. Let us consider two cases in which relation (9a) holds.

1. As soon as the space charge Q_C is formed inside an MHD fast wave or a shock, the electric field due to Q_C is expected to propagate along the magnetic field from the height-limited source region. Such a transmission is carried by an Alfvén wave. Since the source wave (i.e., the MHD fast wave) travels further upstream across the magnetic field before the launched Alfvén wave may be reflected back from, for example, the ionosphere, the reflected wave can be ignored here. The Alfvén wave can carry a certain pair of $J_\|$ and Q_C, which satisfies

$$\begin{aligned}\nabla_\perp\cdot\mathbf{J}_\perp &= \mp\frac{\Sigma_A}{h}\nabla_\perp\cdot\mathbf{E}_\perp\\ &= \mp\sigma_A\nabla_\perp\cdot\mathbf{E}_\perp\end{aligned} \tag{9b}$$

where $\Sigma_A = (\mu_0 V_A)^{-1}$ is the equivalent "conductivity (admittance)"of the Alfvén wave [*Sato and Iijima*, 1979; *Kan and Sun*, 1985], $\sigma_A = \Sigma_A/h$, and V_A is the Alfvén velocity. The minus sign is to be adopted since the current flows out of the plus charge.

2. If the wave is more or less standing in a mixture of plasma which is composed of a preexisting stagnant (or slowly moving) part and an inflow part as shown in Figure 1b, the reflected wave may travel through the stagnant medium and come back to the original position. In this quasi-standing situation, we expect the **J**-**E** relation to be governed by Ohm's law where the wave is reflected back. For example, if the main part of the wave is reflected at the dayside ionosphere where we can assume uniform conductivity (e.g., at dayside cusp), the relation between $\nabla_\perp \cdot \mathbf{J}_\perp$ and $\nabla_\perp \cdot \mathbf{E}_\perp$ can be

$$\begin{aligned} \nabla_\perp \cdot \mathbf{J}_\perp &= -\frac{1}{h} \nabla_\perp \cdot {\mathbf{I}_\perp}^{(i)} \\ &= -\frac{\Sigma_P}{h} \nabla_\perp \cdot {\mathbf{E}_\perp}^{(i)} \\ &= -\sigma_P \nabla_\perp \cdot \mathbf{E}_\perp \end{aligned} \tag{9c}$$

where $\sigma_P = \Sigma_P/h$, and ${\mathbf{I}_\perp}^{(i)}$ and ${\mathbf{E}_\perp}^{(i)}$ are the height-integrated ionospheric current and electric field, respectively. Equation (9c) is exactly the same as equation (9b) except for σ values.

Let us rewrite equation (8) using relation (9a):

$$(\frac{d}{dt} + \frac{1}{\tau}) \frac{\nabla_\perp \cdot \mathbf{E}_\perp}{B^2} = \frac{d}{dt} \frac{\mu_0 \mathbf{J}_\perp \cdot \mathbf{u}_\perp}{B^2} + \alpha \frac{\mathbf{J}_\perp \cdot \nabla B}{\rho B} \tag{10}$$

where $\tau = \rho/\sigma B^2$, which is given as

$$\tau_A \sim \frac{\rho h}{\Sigma_A B^2} \tag{11a}$$

for the first case based on equation (9b), or

$$\tau_P \sim \frac{\rho h}{\Sigma_P B^2} \tag{11b}$$

for the second case based on equation ($9c$). Readers may refer to Figure $3b$ of *Sato and Iijima* [1979] for this case. The decay time τ given in ($11a$) represents how efficiently the electromagnetic energy is taken away by the Alfvén wave, whereas τ_P of ($11b$) represents the energy loss rate due to Joule dissipation in the ionosphere, and of course $\tau_P > \tau_A$ must be satisfied (otherwise, we may not use τ_P for τ). These energy loss rates easily satisfy the relation:

$$|\frac{d}{dt}| > \frac{1}{\tau} \tag{12}$$

because $|d/dt| \sim |V_F \nabla| > V_A L^{-1} > V_A h^{-1} \sim \tau_A^{-1}$, where $V_F = (C_S^2 + V_A^2)^{0.5}$ is the MHD fast velocity across the magnetic field, C_S is the sound speed, and L is the thickness of the wave front.

Equation (10) states how the space charges, and hence the FACs are formed when the fluid element experiences a change of $\mathbf{J}_\perp \cdot \mathbf{u}_\perp$ while $\mathbf{J}_\perp$ experiences a change of B inside the MHD fast wave or shock. The source energy for this charge separation is the kinetic energy of the inflow convection, and this is a kind of dynamo equation. Therefore a convecting medium is required for equation (10) or (8). In fact, the $\mathbf{J}_\perp \cdot \mathbf{u}_\perp$ term becomes zero for the linear approximation (i.e., both $\mathbf{u}$ and $\mathbf{J}$ are first-order small quantities) and in such case, the assumptions ($5a$) and ($5b$) must be reexamined.

Let us evaluate the source terms (right-hand side) of equation (10). Since the full time derivative is approximately $d/dt \sim V_F L^{-1}$, the first source term is evaluated as $(uV_F V_A^{-2}) J \rho^{-1} L^{-1}$ while the second source term is evaluated as $J\rho^{-1}L^{-1}$. Therefore the first source term is dominating as long as

$$\kappa = (\frac{\beta}{2} + 1)\frac{u}{V_F} \tag{13}$$

is large, where β is the ratio of plasma pressure to the magnetic pressure, i.e., $\beta = 2C_S^2 V_A^{-2}$. We now consider a finite convection in which $\kappa > 1$. This condition is rather easily achieved for high β plasma. Under $\kappa > 1$ condition, we may neglect the second source term; i.e., we may safely assume

$\alpha = 0$ without losing generality regardless of the discussions in section 2. Tracing back the $\alpha = 0$ condition to equation (6), one can see that $\kappa > 1$ guarantees an equivalence between $J_{\|}$ and time derivative of the vorticity.

4. Double Field-Aligned Current Structure

We hereafter assume $\kappa > 1$ and hence $\alpha = 0$ in order to extract the most important feature of equation (10) for MHD fast waves or shocks. According to equation (10) under $\alpha = 0$ condition, the distribution of the space charge $Q_C \propto \nabla_\perp \cdot \mathbf{E}_\perp$ can be obtained inside the compressional MHD fast waves or shocks in the configuration of Figure 2*a*. The discussion is given in two stages. We first ignore the magnetic field curvature ($\nabla \times \hat{\mathbf{b}} = 0$), an effect to be evaluated later.

Without Curvature: Two-Dimensional Approximation

Let us ignore the magnetic field curvature ($\nabla \times \hat{\mathbf{b}} = 0$) and extract the most important feature. Omission of the magnetic curvature implies a two-dimensional approximation (perpendicular to the magnetic field) for the wave itself, i.e., plane wave approximation inside the source region. Under this assumption, $\mathbf{u}_\perp \cdot \nabla \times \mathbf{B}$ is rewritten as $(\mathbf{u}_\perp \times \nabla B)_{\|}$, and hence equation (10) is simplified as

$$(\frac{d}{dt} + \frac{1}{\tau})\ \frac{\nabla_\perp \cdot \mathbf{E}_\perp}{B^2} = \frac{d}{dt}\ \frac{(\mathbf{u} \times \nabla B)_{\|}}{B^2} \tag{14}$$

where we already assumed $\alpha = 0$ as discussed above. The profile of the source term is sketched in Figure 2*b*. Since the source term is a full time derivative of $\mathbf{u} \times \nabla B$, it must give a pair of positive and negative values for the same fluid element. Therefore we have a pair of plus and minus charges on the same stream line as a solution of (14). This paired-charge solution is obtained in an MHD scheme, not in a two-fluid scheme. Therefore these charges are not the ordinary (Langmuir type) polarization charges due to finite Larmor radius which are inherent to compressional waves. In fact, no charge is obtained as a solution of (14) if the wave normal is aligned with the convection, while the charges due to the finite Larmor radius must appear in this special situation.

Figure $2a$ illustrates these charges and back-up longitudinal electric current from minus charges to the plus charges as well. Note that there is another current which lies along the wave front. This transversal current is always found in the MHD fast waves or shocks, and is normally stronger than the longitudinal current; however, it does not contribute to the charge separation given here. The senses of the paired-charges depend on the wave normal direction with respect to the convection direction. Once a pair of charges are formed, relation $(9a)$ guarantees a generation of a pair of FACs which is closed with the polarization current inside the wave. This double field-aligned current structure is the most distinct characteristics of the field-aligned current generation from a compressional MHD fast wave or shock. There is another characteristics: asymmetry on the charge distribution is introduced by the presence of the decay term on the left-hand side of (14).

Effect of Curvature

Next, we examine if the double current structure still exists when the curvature effect is included. In order to evaluate the source terms of equation (10), we need to know the current distribution by integrating $(9a)$:

$$\mathbf{J}_\perp = \sigma(\mathbf{u}_\perp \times \mathbf{B} - \mathbf{u}_{in} \times \mathbf{B}_{in}) + \mathbf{J}_0 \qquad (15)$$

where subscript “in”denotes an upstream value and $\mathbf{J}_0$ is an integration constant, which must be a divergence-free vector. Strictly speaking, a simulation based on at least the two-fluid theory is necessary to know the actual current distribution, or $\mathbf{J}_0$. However, we here assume a constant and uniform $\mathbf{J}_0$ in the $\mathbf{u}_{in} \times \mathbf{B}_{in}$ direction as the most simple form for $\mathbf{J}_0$. This simplification is rational because the present purpose is to check whether we may still have a paired-charge solution even if the magnetic field curvature is taken into account.

Substituting (15) into (10) under $\alpha = 0$ condition, one obtains

$$(\frac{d}{dt} + \frac{1}{\tau})\ \frac{\nabla_\perp \cdot \mathbf{E}_\perp}{B^2} = \mu_0(\mathbf{J}_0 - \sigma \mathbf{u}_{in} \times \mathbf{B}_{in}) \cdot \frac{d}{dt}\ \frac{\mathbf{u}_\perp}{B^2} \tag{16}$$

As long as $|\sigma u B| > J_0$, the profile of the source term of (16) in the Figure $2a$ configuration is morphologically the same as what we have obtained already, and hence, the paired-charge solution is again obtained.

Let us mention a perfect two-dimensional case (source region is not limited along the magnetic field). In this case, the spatial derivative along the magnetic field must be zero leading to $\sigma = 0$ in equation $(9a)$. Thus we have only a plus or a minus value for the solution of (8). In other words, we need a limited height of the source region along the magnetic field in order to have the double field-aligned current structure. This is another evidence that the paired-charge solution obtained here is different from a Langmuir type longitudinal polarization typical for compressional MHD fast waves or shocks.

5. Conclusions and Possible Applications

We have obtained a simplified form of the vorticity equation (6) for the MHD fast waves or shocks when conditions $(5a)$ and $(5b)$ are satisfied. In a special configuration such as of Figure 1 when equation $(9a)$ is also satisfied, the vorticity equation can be rewritten as in (10). A pair of FACs can be generated due to changes of $\mathbf{u}_\perp \cdot \mathbf{J}_\perp$ depending on the wave normal direction with respect to a finite background convection as is demonstrated in Figure 2 with equation (14). This wave converts the kinetic energy of the inflow convection into the electromagnetic energy of the FACs. Thus equation (10) can also be interpreted as a dynamo equation. Contributions from the curvature drift, which we believe is only a minor contributer to the vorticity equation for the configuration shown in Figure 1, can be substantially neglected if $\kappa > 1$, i.e., if the plasma β is high or the convection is fast enough. In the linear limit when $\kappa < 1$, i.e., when we do not have the source energy in the form of finite convection, the present formulation does not apply. The resultant paired-charge structure is different from the ordinary (Langmuir type) longitudinal polarization which is inherent to a compressional wave because the present result is formulated in the MHD scheme.

As possible applications, the authors propose two places in the magnetosphere: 1. the dayside cusp where the solar wind may directly enter the exterior cusp and become decelerated to produce the cusp region 1 FAC and the mantle FAC [e.g., *Erlandson et al.*, 1988] as shown in Figure 3*a*; and 2. the nightside where the substorm current wedge may propagate tailward against the convection, causing dipolarization [*Lopez and Lui*, 1990; *Yamauchi*, 1990] as shown in Figure 3*b*. Equation (16) shows that the double current structure is expected even if there is a background current in the upstream region. There are several reports about such double field-aligned current structures inside expanding auroral arcs during the substorm expansive phase [*Zanetti and Potemra*, 1992; *Yamauchi et al.*, 1992]. The initial excitation of this wave (increase of pressure) may be caused by direct compression as a result of enhanced convection. One of several possible mechanisms for enhanced convection is given by *Lui et al.* [1991, 1993] in which a force imbalance led by an instability (such as the cross-field instability) accelerates plasma to high speeds while causing a current disruption in addition. Another mechanism is simply local enhancements of the tail polarization electric field, or convection, driven by pressure irregularity in the low-latitude boundary layer [*Lundin et al.*, 1992]. Further studies are of course necessary to clarify these applications.

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Fig. 1. Configurations under consideration. There exists the MHD (a) fast wave or (b) fast shock with extent limited in the direction of the magnetic field. Since we mostly consider the geospace environment, the magnetic field converges outside the source region. The background plasma is moving toward the wave or the shock (κ defined in equation (13) is larger than unity), and for Figure 1b at a supersonic speed. When particles which experience the curvature of the magnetic field inside the wave are reflected back from, for example, the magnetic mirror carrying the information of the "drift,"the wave itself is already far upstream of its original position, and the reflected particles cannot catch up with the wave.

Fig. 2. (a) Space charge separation inside a nonlinear MHD fast wave propagating against a finite velocity convection is illustrated. The convection is decelerated across the wave giving its kinetic energy to the current system. Therefore the polarization current flows from negative to positive. The polarity of the FAC is very sensitive to the direction of the wave normal with respect to the convection direction. (b) These space charges are calculated based on equation (14). The trailing charges in the downstream region arise from the decay (second) term on the left-hand side of the equation.

Fig. 3. Possible magnetospheric regions where the double field-aligned current structure could be found due to the present mechanism: (a) dayside cusp where the magnetosheath flow is supposed to be supersonic [e.g., *Walters*, 1966]; and (b) substorm current wedge travelling tailward against convection. The senses of the separated space charges are obtained based of the directions of the geomagnetic field, flow, and the wave normal. In both cases, the surface current which is inherent to MHD fast waves is terminated at both dawn and dusk sides because the dawn-dusk extent of the wave is also limited. This current accumulates charges on the front side of the wave. In the night side, there is also a cross tail current in the opposite direction of this surface current, and as a result, the double field-aligned current system is generated where the cross-tail current is disrupted.

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