Wavelet Based Methods for Time Dependent PDEs

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... *Time Dependent PDEs*

Partial differential equations, e.g., the Navier-Stokes equations in two dimensions:

$$
\left\{
\begin{array}{lcl}
\rho_t & = & -\rho_x u - \rho_y v - \rho(u_x + v_y) \\
u_t & = & -u u_x - v u_y + \frac{1}{\rho \mathrm{Re}} \left((2\mu + \lambda) u_{xx} \right. \\
 & & \left. +(\mu + \lambda) v_{xy} + \mu u_{yy} \right) \\
 & & -\frac{R}{\rho}(\rho_x T + \rho T_x) \\
v_t & = & -v v_y - u v_x + \frac{1}{\rho \mathrm{Re}} \left((2\mu + \lambda) v_{yy} \right. \\
 & & \left. +(\mu + \lambda) u_{xy} + \mu v_{xx} \right) \\
 & & -\frac{R}{\rho}(\rho_y T + \rho T_y) \\
T_t & = & -u T_x - v T_y - (\gamma - 1)(u_x + v_y) T \\
 & & +\frac{\gamma k}{\rho \mathrm{Re} \mathrm{Pr}} (T_{xx} + T_{yy}) \\
 & & +\frac{\gamma - 1}{\rho \mathrm{Re} R} \left(\lambda (u_x + v_y)^2 + \mu (u_y + v_x)^2 \right. \\
 & & \left. +2\mu (u_x^2 + v_y^2) \right)
\end{array}
\right.
$$

Discretization

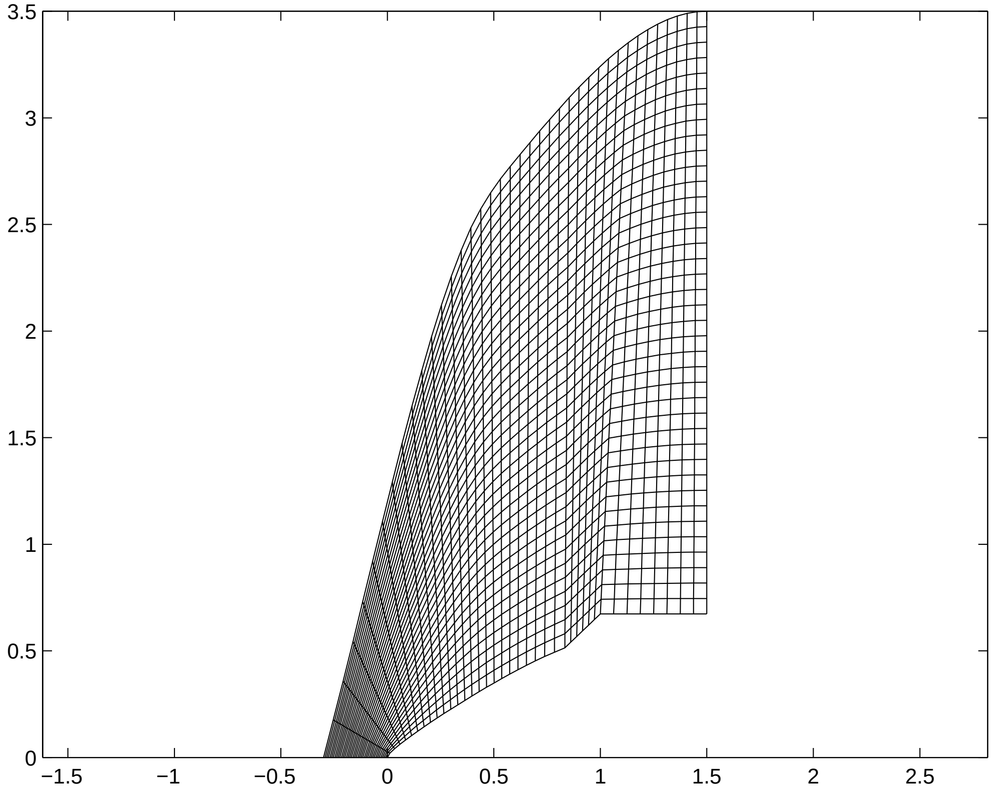

Figure 1: Two-dimensional grid

Approximate Solution

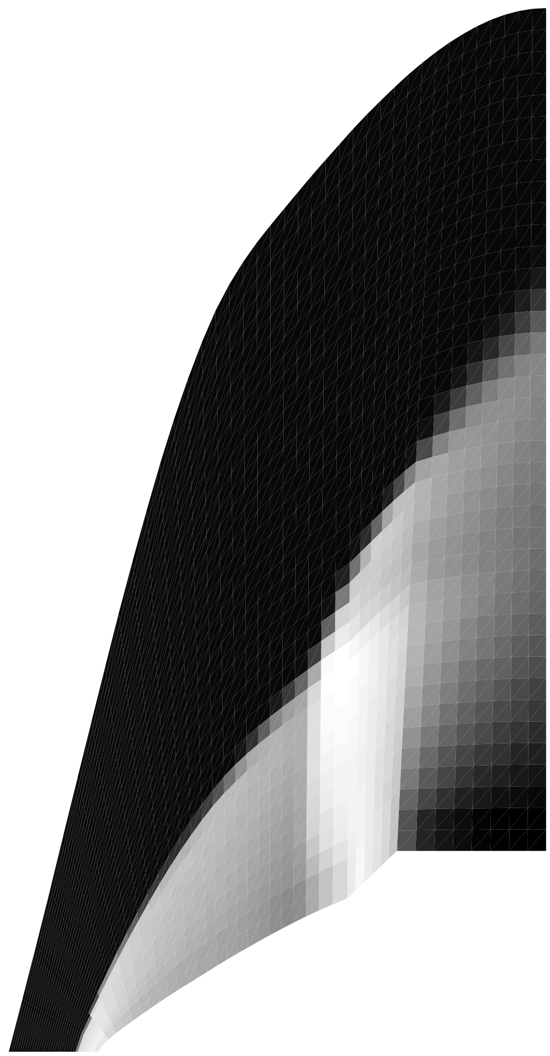

Figure 2: Temperature

Wavelet Based Methods . . .

Position

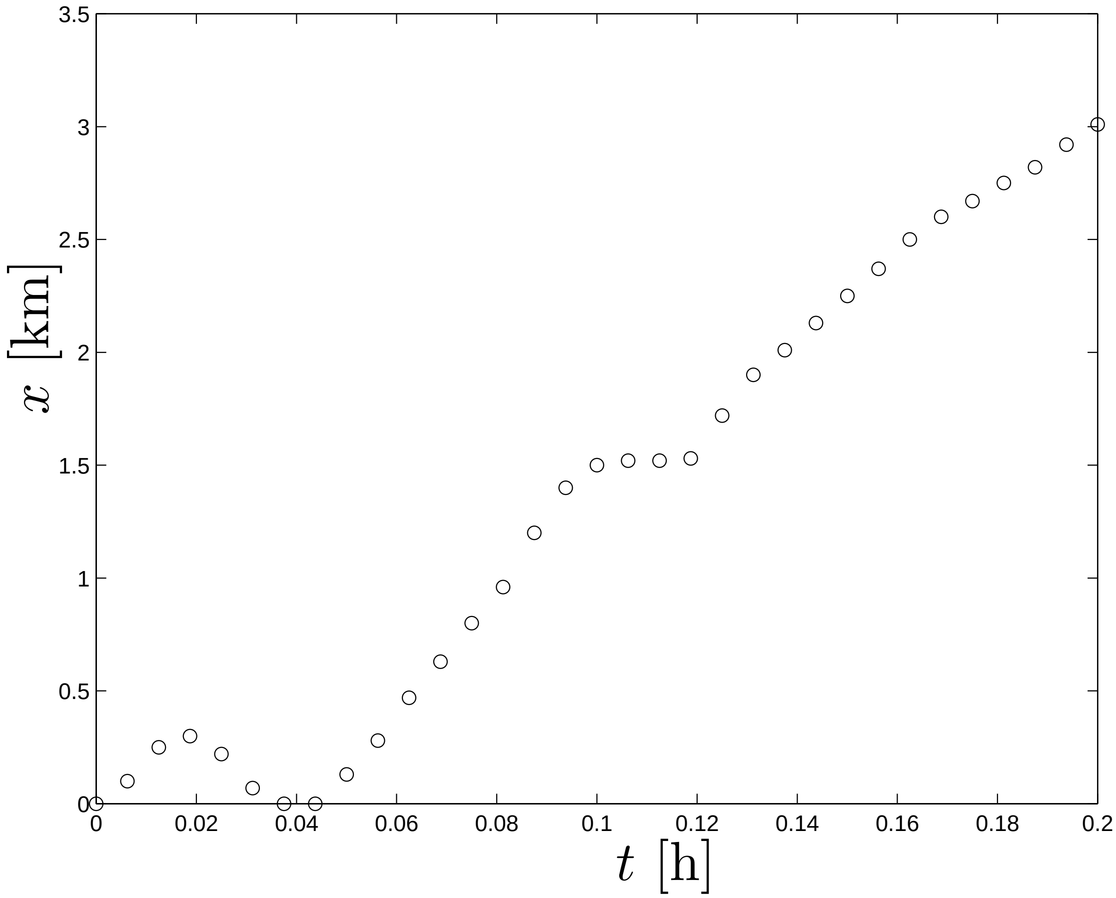

Figure 3: Position as a function of time

Differentiation

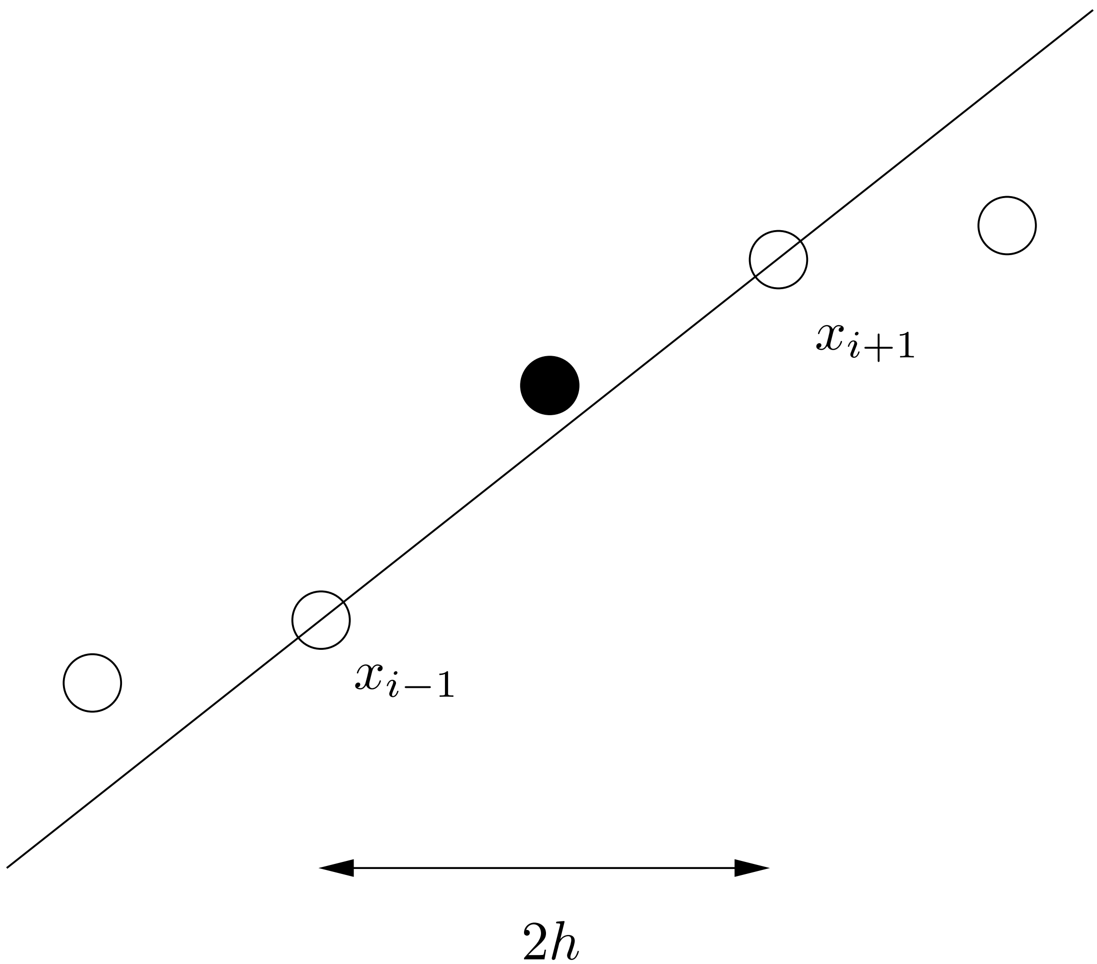

Figure 4: Approximation of the velocity

$$v \approx \frac{x_{i+1} - x_{i-1}}{2h}$$

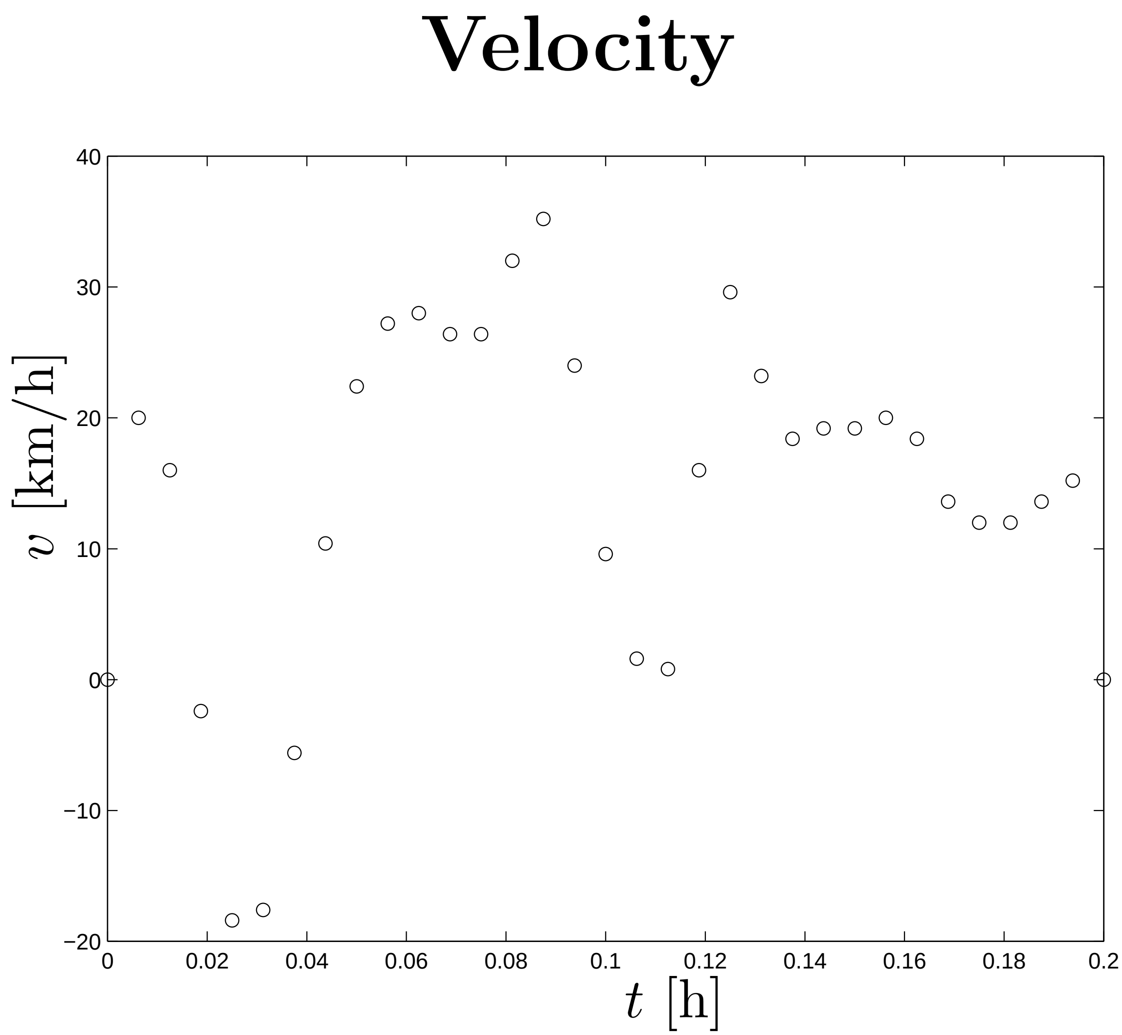

Figure 5: Velocity at all points

Interpolation

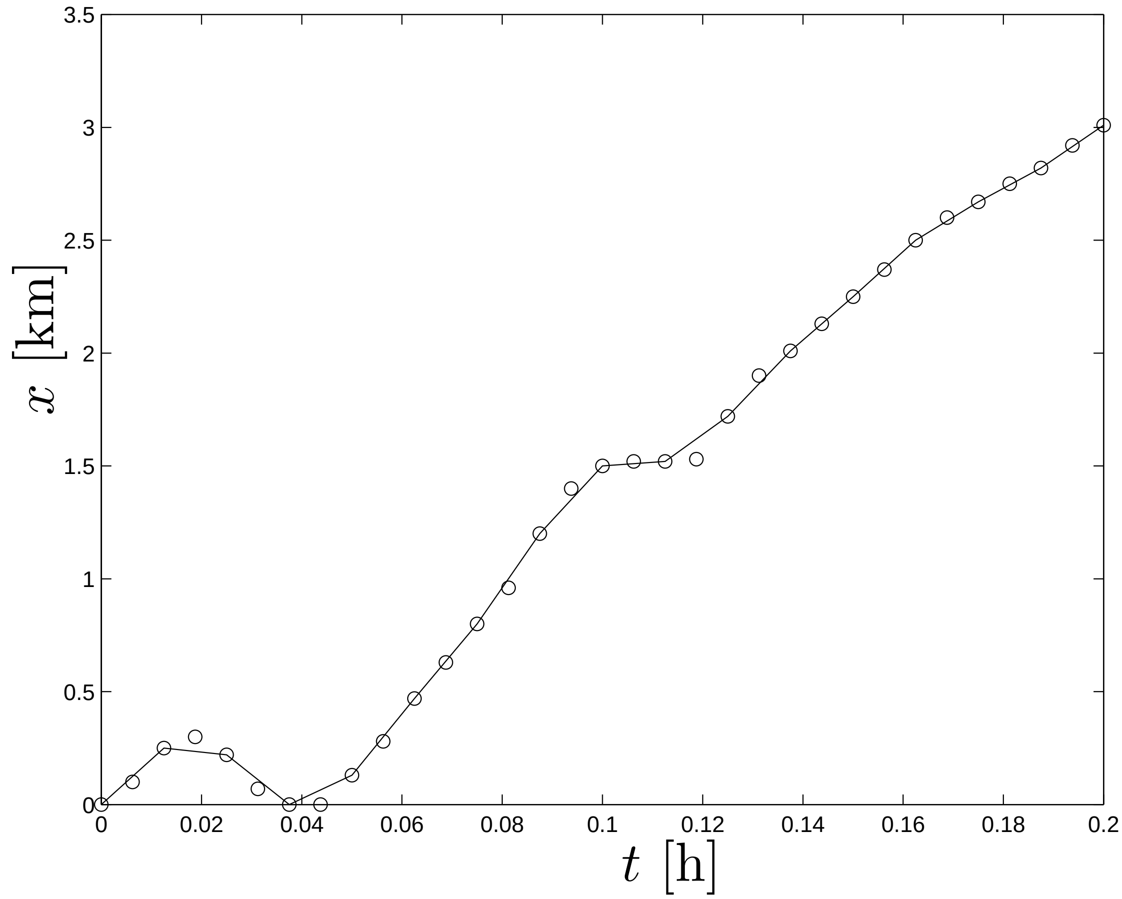

Figure 6: Skip every other point

Thresholding

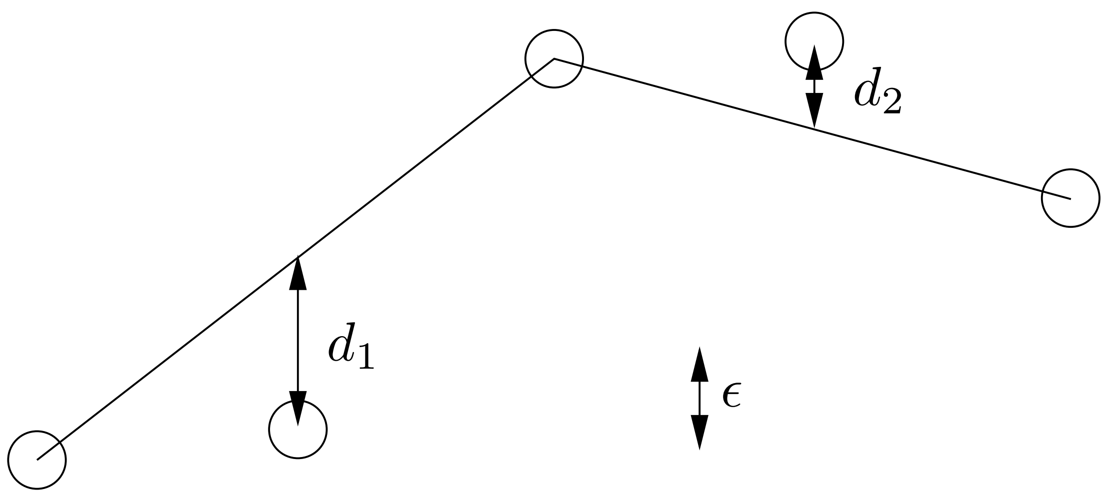

Figure 7: Remove point if $|d_i| < \epsilon$

Sparse Representation

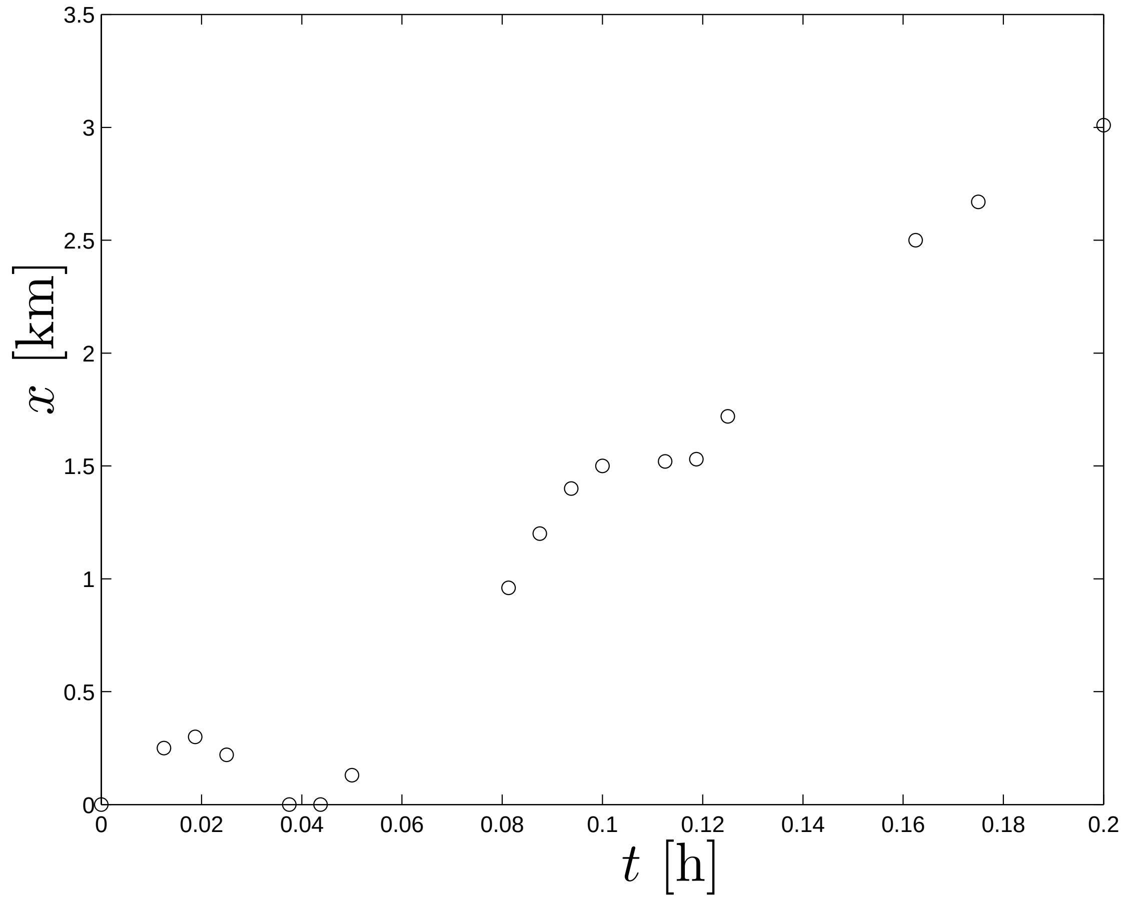

Figure 8: Sparse point representation

Reconstruction

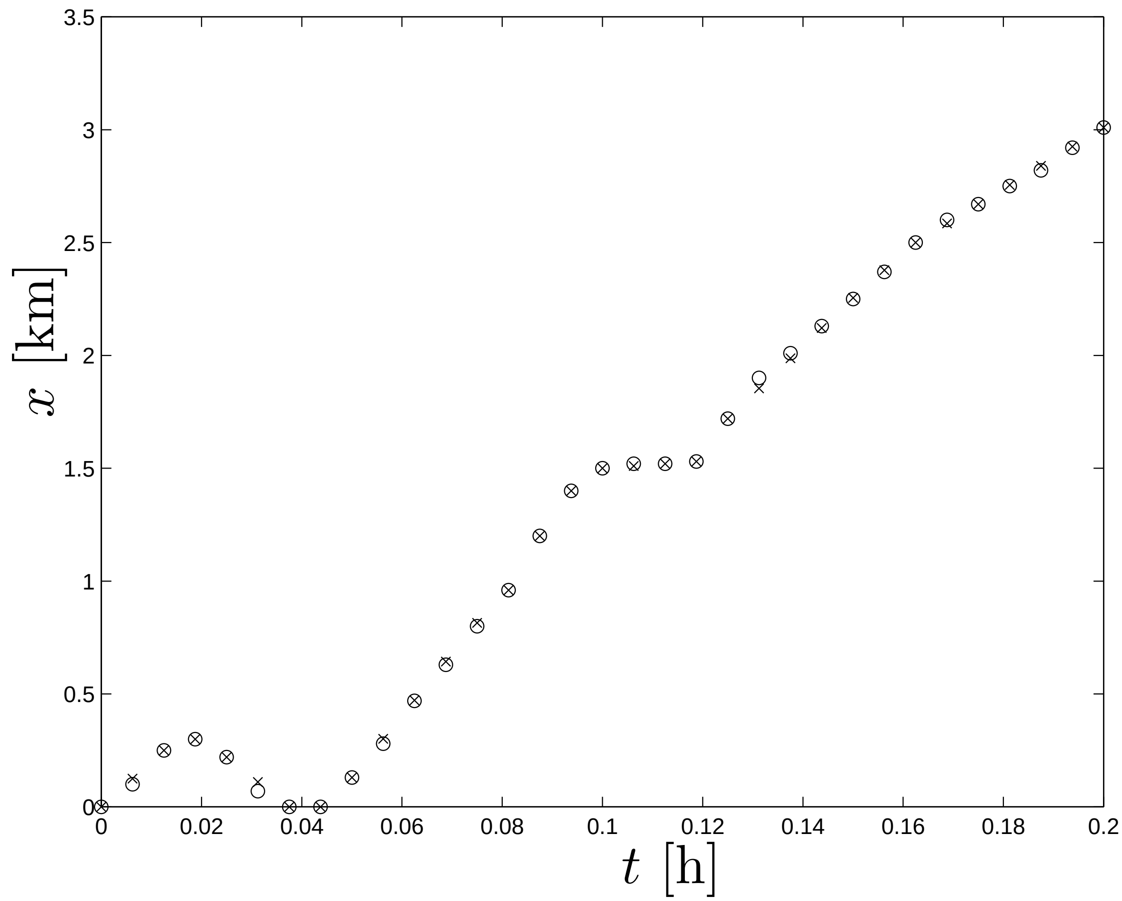

Figure 9: Reconstructed points

Sparse Velocity

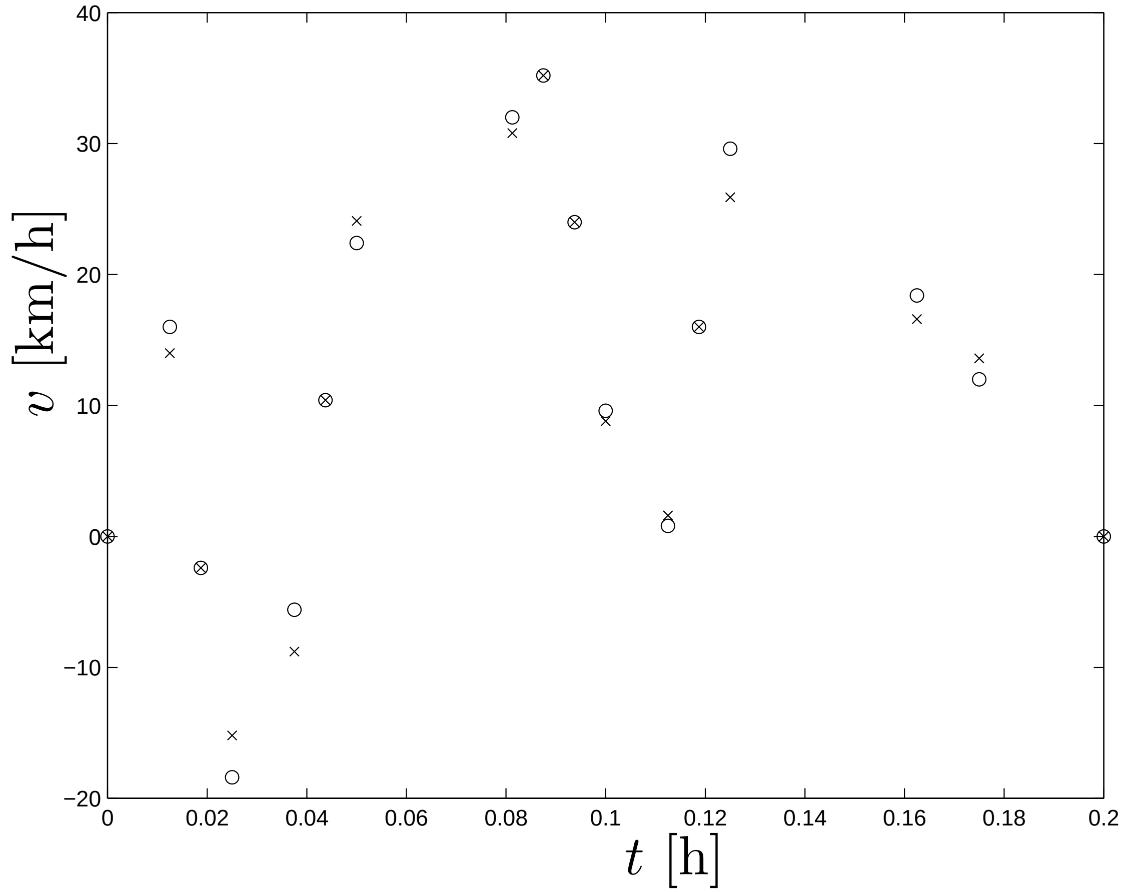

Figure 10: Velocity at the remaining points

Conclusions

A method for solving PDEs that saves

- computational time
- memory requirements