Wavelet Based Methods
for
Time Dependent PDEs

Mats Holmström
\[ \begin{align*}
\rho_t &= -\rho_x u - \rho_y v - \rho (u_x + v_y) \\
\rho \frac{u_t}{\rho} &= -uu_x - v u_y + \frac{1}{\rho \text{Re}} ((2\mu + \lambda)u_{xx} \\
&\quad + (\mu + \lambda)v_{xy} + \mu u_{yy}) \\
&\quad - \frac{R}{\rho}(\rho_x T + \rho T_x) \\
\rho \frac{v_t}{\rho} &= -vv_y - u v_x + \frac{1}{\rho \text{Re}} ((2\mu + \lambda)v_{yy} \\
&\quad + (\mu + \lambda)u_{xy} + \mu v_{xx}) \\
&\quad - \frac{R}{\rho}(\rho_y T + \rho T_y) \\
\rho \frac{T_t}{\rho} &= -uT_x - vT_y - (\gamma - 1)(u_x + v_y)T \\
&\quad + \frac{\gamma k}{\rho \text{RePr}} (T_{xx} + T_{yy}) \\
&\quad + \frac{\gamma - 1}{\rho \text{RePr}} (\lambda (u_x + v_y)^2 + \mu (u_y + v_x)^2 \\
&\quad + 2\mu(u_x^2 + v_y^2))
\end{align*} \]
Discretization

Figure 1: Two-dimensional grid
Approximate Solution

Figure 2: Temperature
Wavelet Based Methods . . .

Position

Figure 3: Position as a function of time
Differentiation

Figure 4: Approximation of the velocity

\[ v \approx \frac{x_{i+1} - x_{i-1}}{2h} \]
Figure 5: Velocity at all points
Interpolation

Figure 6: Skip every other point

\[ x \text{ [km]} \]
\[ t \text{ [h]} \]
Thresholding

Figure 7: Remove point if $|d_i| < \epsilon$
Sparse Representation

Figure 8: Sparse point representation
Reconstruction

Figure 9: Reconstructed points
Figure 10: Velocity at the remaining points
Conclusions

A method for solving PDEs that saves

- computational time
- memory requirements